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Approximation Formula For The Prediction Of Downwind Distance That Found The Maximum Ground Level Concentration Of Air Pollution Based On The Gaussian Model

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Abstract

This research developed an approximation formula for the prediction of downwind location from a stack that found the maximum ground level concentration of air pollution based on Gaussian Model using Brigg Method for the calculation of σ_y and σ_z. The estimated and actual x_{max} were summarized. The errors were in the range of 0 to 2713.3%. The high accuracy of the estimation would likely be observed at the lower x_{max} value and vice versa. However, this technique has a limitation for the stability classes of E and F for the open-country environments as that approximation method could be used for the stack height shorter than 141.4 m and 7.54 m, respectively.

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Keywords: Air pollution; plume distribution; modeling; continuous point source

1. Introduction

Air pollution is one of the major problems in environmental science. It is necessary to evaluate the environmental impact from projects that we are going to do to prevent adverse effects to the environment. The mathematical model is a very essential tool for environmental impact assessments especially for the impact from air pollution as it is costly and it can predict the effect before the project is started.

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Nomenclature

x	the downwind distance from the source (m),
y	the lateral distance from the source (m),
z	the vertical distance above the ground (m),
C	the concentration of the pollutant at a point with coordinates x, y, z (g/m^3),
u	the downwind speed (m/s),
q_p	the emission rate (g/s),
H	the effective source height above the ground (m),
σ_y	the plume dispersion parameter in the lateral directions (m),
σ_z	the plume dispersion parameters in vertical directions (m).
GLMC	the ground level maximum concentration of pollutants
x_{max}	the downwind distance that GLMC can be found (m)

The Gaussian Air Pollution Model is one of the most famous mathematical models which can predict the concentration of pollutants downwind from a continuous point source as depicted in Fig. 1.

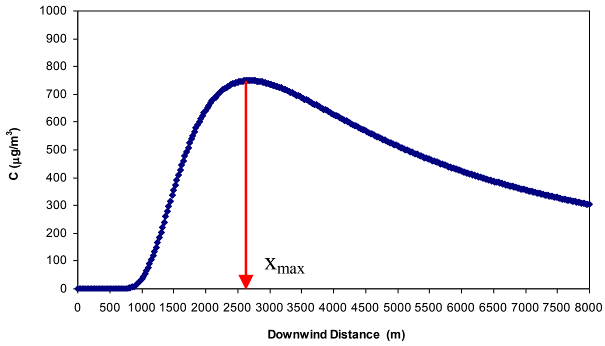

Fig. 1. The graphical plotting to determine actual x_{max} from the Gaussian Model.

The Gaussian Model was derived by many authors (Essa, Etman, & Embaby, 2003) such as Sutton (1953), Turner (1970), Csanady (1973), and Smith (1973). The use of Gaussian Air Pollution Model requires the estimation of horizontal and vertical growth of the plumes for predicting the air pollutant concentration and the horizontal and vertical growth of plumes are generally expressed in terms of standard deviations of concentrations in lateral (y) and vertical (z) directions, i.e., σ_y and σ_z, respectively and parameterize the dispersion due to atmospheric turbulence (Mohan & Siddiqui, 1997). The equation of Gaussian Air Pollution Model can be expressed as Eq. (1) (IAEA Safety guide, 1983):

$$C(x,y,z,H)=\frac{q_p}{2\pi\sigma_y\sigma_z u}e^{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2}\left[e^{-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2}+e^{-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2}\right] \tag{1}$$

The above equation can be reduced to the simple terms for the prediction of concentration at the ground level (by setting z = 0) in the plume centerline (by setting y = 0) as follows:

$$C(x,0,0,H)=\frac{q_p}{\pi\sigma_y\sigma_z u}\left[e^{-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2}\right] \tag{2}$$

The variables σ_y and σ_z are the function of many independent variables such as downwind distance from the source (x variable), downwind speed (u), and the atmospheric stability class.

Many researchers tried to evaluate the σ_y and σ_z as follows: Pasquill and Gifford generated P–G curves for estimating σ_y and σ_z (Gifford, 1961) as a function of downwind distance for the different stability classes. However, it is very difficult to use these curves to predict the pollutant concentration by computer programming. Brigg (1973) proposed the empirical formula to predict the σ_y and σ_z based on the revision of P-G curves. The US-EPA (1977) generated an asymptotic formula for the P-G curves which yields reasonably accurate results; Draxler (1976) and Irwin (1979) independently developed the complicated model to calculate σ_y and σ_z. The comparison between these models shows that the Brigg model is the best fit with observable data (Mohan & Siddiqui, 1977) and its calculation of σ_y and σ_z can be summarized in Tables 1.

The importance of the Gaussian Air Pollution Model is the determination of the ground level maximum concentration (GLMC) of pollutants and the downwind distance that GLMC can be found (x_{max}). This can represent how and where the worst situations that can occur from the pollutant emissions from stacks. The rough estimation for the x_{max} was proposed as the distance that makes σ_z equal to $\frac{H}{\sqrt{2}}$ (Kenneth & Cecil, 1972) and the GLMC can be calculated by substitution of x_{max} in the formula of concentration for the ground level in the plume centerline or Eq (2).

Since Brigg's approximation of σ_y and σ_z is one of the best asymptotic formulas to the observed results (Mohan & Siddiqui, 1997), this article derived the relations to estimate x_{max} based on the Brigg's determination of σ_y and σ_z as shown in Table 1

Table 1 Brigg Model for the prediction of σ_y and σ_z *

Stability Class	the open-country environment		the urban environment	
	σ_y (m)	σ_z (m)	σ_y (m)	σ_z (m)
A (very unstable)	$\frac{0.22x}{\sqrt{1+0.0001x}}$	$0.20x$	$\frac{0.32x}{\sqrt{1+0.0004x}}$	$0.24x\sqrt{1+0.001x}$
B (moderately unstable)	$\frac{0.16x}{\sqrt{1+0.0001x}}$	$0.12x$	$\frac{0.32x}{\sqrt{1+0.0004x}}$	$0.24x\sqrt{1+0.001x}$
C (slightly unstable)	$\frac{0.11x}{\sqrt{1+0.0001x}}$	$\frac{0.08x}{\sqrt{1+0.0002x}}$	$\frac{0.22x}{\sqrt{1+0.0004x}}$	$0.20x$
D (neutral)	$\frac{0.08x}{\sqrt{1+0.0001x}}$	$\frac{0.06x}{\sqrt{1+0.0015x}}$	$\frac{0.16x}{\sqrt{1+0.0004x}}$	$\frac{0.14x}{\sqrt{1+0.0003x}}$
E (slightly stable)	$\frac{0.06x}{\sqrt{1+0.0001x}}$	$\frac{0.03x}{1+0.0003x}$	$\frac{0.11x}{\sqrt{1+0.0004x}}$	$\frac{0.08x}{\sqrt{1+0.0015x}}$
F (stable)	$\frac{0.04x}{\sqrt{1+0.0001x}}$	$\frac{0.016x}{1+0.0003x}$	$\frac{0.11x}{\sqrt{1+0.0004x}}$	$\frac{0.08x}{\sqrt{1+0.0015x}}$

*Modified from Brigg (1973)

2. Material and methods

The formula of x_{max} is derived from the principle of rough estimation by solving equation

$$\sigma_z = \frac{H}{\sqrt{2}} \tag{3}$$

where σ_z was substituted by the relationship expressed in Table 1 and x was substituted with x_{max} then solved for the x_{max} and summarized in Table 2.

According to the Table 1, there are 4 forms of expression for σ_z as follows:

Form I : $\sigma_z = \alpha x$, This can be easily solved for x_{max} and can be expressed in the form of $x_{max} = \frac{H}{\alpha\sqrt{2}} = \frac{H\sqrt{2}}{2\alpha}$

Form II : $\sigma_z = \frac{\alpha x}{1+\beta x}$, This can be solved for x_{max} as $x_{max} = \frac{H}{(\sqrt{2}\alpha - \beta H)}$.

Form III : $\sigma_z = \frac{\alpha x}{\sqrt{1+\beta x}}$, This can be solved for x_{max} using the knowledge of the quadratic equation as

$2\alpha^2x^2-\beta H^2x-H^2=0$.

Since the quadratic equation has 2 answers, the positive real number is selected to be an answer of x_{max}.

Form IV : $\sigma_z = \alpha x\sqrt{1+\beta x}$, This can be solved for x_{max} using the knowledge of the cubic equation.

The Wolfram Alpha algebraic program was used to solve such an equation.

Since the cubic equation has 3 answers, the positive real number is selected to be an answer of x_{max}.

The situation of pollutant's emission is simulated to determine the accuracy of the results. The actual x_{max} of the Gaussian Model according to Sutton (1953) is determined using Microsoft Excel's graphical plotting as a sample shown in Fig. 1. Three effective heights at 5 m, 100 m, and 250 m were evaluated using fixed q_p and u of 3 g/s and 3 m/s, respectively. The comparisons between the actual x_{max} and the estimated values obtained from the Table 2 are also investigated. The percentage of relative error was calculated by

$$\text{Relative error} = \left|\frac{\text{actual } x_{max} - \text{estimated } x_{max}}{\text{actual } x_{max}}\right| \times 100 \tag{4}$$

3. Results and Discussion

3.1. Expression for determination of x_{max}

The formula for the estimation of x_{max} is summarized in Table 2. It is interesting to note that the expression of x_{max} for stability classes A and B of the urban environment is very complicated. This is because the original equation is the cubic equation (3 degrees of polynomial equation). To reduce such difficulty, the regression technique could be applied to make it simpler. The expression can be simplified as follow

$$x_{max} \approx 2.658\,H - 0.0012\,H^2 \tag{5}$$

The Eq. (5) is created on the basis of the effective height from 0 to 500 m with the pinch of 10 m (0, 10, 20, … , 500). The equation fits very well with values obtained from the complicated equation as its correlation coefficient (R^2) is very close to the unity (R^2 = 0.9997).

TABLE 2 CALCULATION OF X_{MAX}

Stability Class	x_{max} in the open country environment	x_{max} in the urban environment
A	$\frac{H\sqrt{2}}{0.4}$	$\frac{25}{3}\left(\sqrt[3]{\frac{5}{2}}\sqrt[3]{3H^2+\sqrt{3}\sqrt{3H^4-51200H^2}-25600}+\frac{320\sqrt[3]{2}\,5^{2/3}}{\sqrt[3]{3H^2+\sqrt{3}\sqrt{3H^4-51200H^2}-25600}}-40\right)$
B	$\frac{H\sqrt{2}}{0.24}$	$\frac{25}{3}\left(\sqrt[3]{\frac{5}{2}}\sqrt[3]{3H^2+\sqrt{3}\sqrt{3H^4-51200H^2}-25600}+\frac{320\sqrt[3]{2}\,5^{2/3}}{\sqrt[3]{3H^2+\sqrt{3}\sqrt{3H^4-51200H^2}-25600}}-40\right)$
C	$\frac{H}{128}(H+\sqrt{H^2+1280000})$	$\frac{H\sqrt{2}}{0.4}$

D	$\frac{5H}{48}(H+\sqrt{H^2+12800})$	$\frac{H}{784}(3H+\sqrt{9H^2+15680000})$
E	$\frac{10000H}{3(100\sqrt{2}-H)}$	$\frac{5H}{256}(3H+\sqrt{9H^2+204800})$
F	$\frac{10000H}{(16\sqrt{2}-3H)}$	$\frac{5H}{256}(3H+\sqrt{9H^2+204800})$

3.2. The impacted parameters on x_{max}

The prediction of x_{max} based on these input variables is summarized in the Tables 3 and 4. It can be deducted from the tables that x_{max} is dependent on the stability class and the effective height of the pollution source. It can be observed from the tables that more atmospheric stability results in both actual and estimated x_{max} being farther. The same trend was found for the effective height (H) as the higher H results in the longer x_{max}.

There were some anomalies found from the stability classes of E and F for the rural area (open-country environment) such as when H is higher than $100\sqrt{2}$ m (roughly 141.4 m) and $16\sqrt{2}/3$ (about 7.54 m); it will give negative results for the estimated x_{max} for the stability class of E and F, respectively. This is the limitation of the rough estimation by assuming x_{max} occurs at σ_z equal to $H/\sqrt{2}$.

Table 3 Accuracy for the Calculation of x_{max} in the open-country environment

Stability Class	H = 5 m Actual x_{max} (m)	H = 5 m Estimated x_{max} (m)	H = 5 m Relative Error (%)	H = 100 m Actual x_{max} (m)	H = 100 m Estimated x_{max} (m)	H = 100 m Relative Error (%)	H = 250 m Actual x_{max} (m)	H = 250 m Estimated x_{max} (m)	H = 250 m Relative Error (%)
A	18	18	0	355	354	0.3	893	884	1.0
B	29	29	0	593	589	0.7	1498	1473	1.7
C	44	44	0	955	965	1.0	2684	2751	2.5
D	61	62	1.6	2194	2615	19.2	11533	13656	18.4
E	121	122	0.8	4559	8047	76.5	32290	-7675	123.8
F	233	6555	2713.3	16433	-3605	121.9	134303	-3437	102.6

Table 4 Accuracy for the Calculation of x_{max} in the urban environment

Stability Class	H = 5 m Actual x_{max} (m)	H = 5 m Estimated x_{max} (m)	H = 5 m Relative Error (%)	H = 100 m Actual x_{max} (m)	H = 100 m Estimated x_{max} (m)	H = 100 m Relative Error (%)	H = 250 m Actual x_{max} (m)	H = 250 m Estimated x_{max} (m)	H = 250 m Relative Error (%)
A	15	15	0	271	262	3.3	618	585	5.3
B	15	15	0	271	262	3.3	618	585	5.3
C	18	18	0	359	354	1.4	915	884	3.4
D	25	25	0	548	545	0.5	1542	1524	1.2
E	44	46	4.5	966	1646	70.4	2715	7939	192.4
F	44	46	4.5	966	1646	70.4	2715	7939	192.4

3.3. Accuracy of the results

The accuracy of estimated x_{max} is summarized in Tables 3 and 4. The errors are in the range of 0 to 2713.3% Generally, the high accuracy for estimated x_{max} would likely be observed at the lower x_{max} value. The higher effective height would give a higher error for the the estimation of x_{max} except in stability class F for the open-country environment. This is because the limitation of the formula in that it could predict x_{max} for the effective height less than 7.54 m.

4. Conclusion

The developed formula in research could be used in the prediction of the location which found the maximum ground level concentration of the emitted air pollutant from the stack or x_{max}. This could be consequently used in the prediction of the maximum ground level concentration which is very useful for the various environmental fields eg. Environmental and Health Impact Assessment (EHIA). There are some limitations to apply such a technique with the high stack effective height, especially for atmospheric stability classes of E and F, which required the further development to correct such a problem.

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Reference

Briggs, G. A. (1973). Diffusion Estimation for Small Emissions. *Atmospheric Turbulence and Diffusion Laboratory contribution.* File # 79.
Csandy, G.T. (1973). Turbulent diffusion in the environment. *Dordrecht: North-Holland*.
Draxler, R. R. (1976). Determination of Atmospheric Diffusion Parameters. *Atmospheric Environment, 10*, 363–372.
EPA. (1977). User's Manual for Single Source (CRSTER) Model. *EPA-450/2-77-013*.
Essa, K. S. M., Etman, S. M., & Embaby, M. (2003). Air pollutant concentration and its location under different thermal stability classes. *Meteorology and Atmosphere Physics, 83*, 123–129.
Gifford, F. A. (1961). Use of Routine Meteorological Observations for Estimating Atmospheric Dispersion. *Nuclear Safety, 2(4)*, 47–51.
IAEA Safety Guide. (1983). IAEA # 50-5G-S3
Irwin, J. S. (1979). Schemes for Estimating Dispersion Parameters as a Function of Release Height. *EPA-600/4-79-062*.
Kenneth, W., & Cecil, F.W. (1972). Air Pollution Its Original and Control. Adun-Donnelly Publisher, New York.
Mohan, M., & Siddiqui M. (1997). An evaluation of dispersion coefficients for use in air quality models. *Boundary-Layer Meteorology, 84*, 177–206.
Smith, M.E. (1973). Recommended guide for the prediction of the dispersion of airborne effluents. ASME # 68-31123.
Sutton, O.G. (1953). Micrometeorology. Quarterly Journal of the Royal Meteorological Society. 79(341), 457.
Turner D.B. (1970). Workbook of atmospheric dispersion. *U.S. Environment Protection Agency*, AP 26.